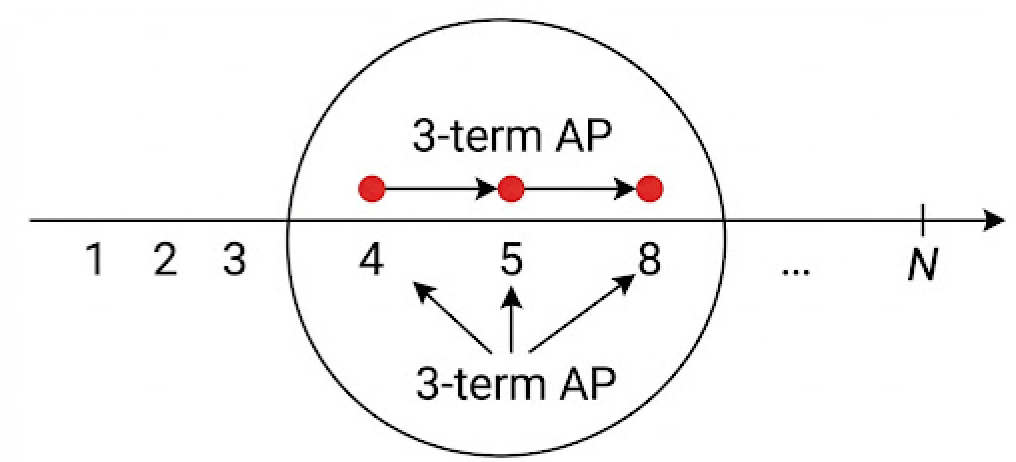
Step 1: What is a k-term Arithmetic Progression?

An arithmetic progression is a sequence of numbers where the difference between consecutive terms is constant. For example, 3, 5, 7 is a 3-term arithmetic progression with a common difference of 2. The problem is about finding subsets of numbers that *avoid* these patterns.

The image below shows a set of numbers where a 3-term arithmetic progression (4, 5, 8 is not an AP, but 2, 5, 8 or 4, 6, 8 would be) is present. The goal is to build a set without any such pattern.

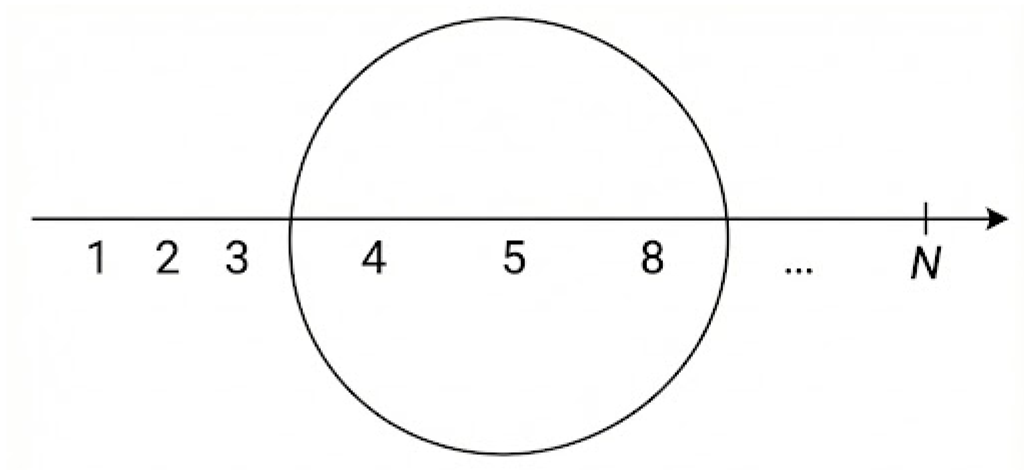


A set with a 3-term AP

Step 2: What is $r_k(N)$?

The quantity $r_k(N)$ is defined as the size of the **largest possible subset** of the numbers $\{1, 2, ..., N\}$ that does **not** contain any k-term arithmetic progression.

The image below illustrates this concept. It shows a set of numbers (4, 5, 8) that do not form a 3-term AP. The size of the largest such set you can find is the value of $r_k(N)$.

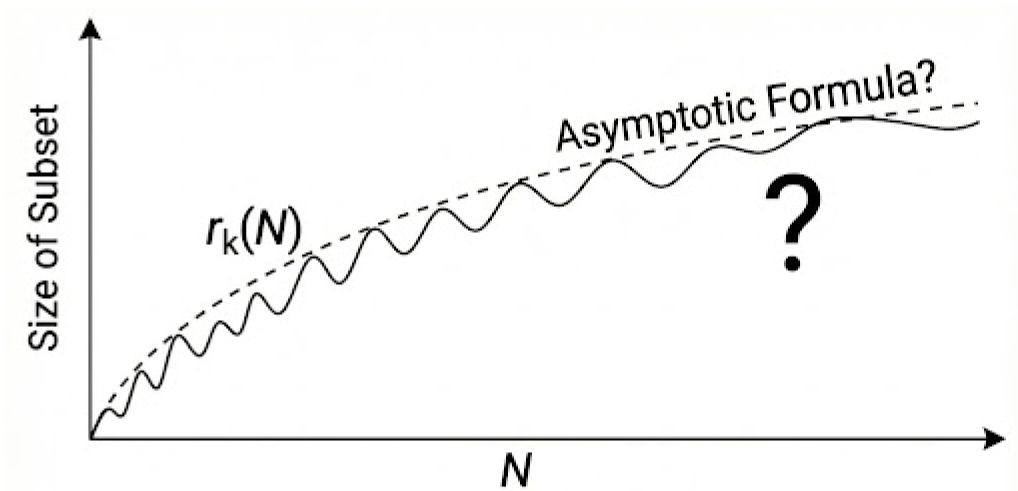


The largest subset without a 3-term AP, size = $r_3(N)$

Step 3: The Goal - An Asymptotic Formula

The problem asks for an **asymptotic formula** for $r_k(N)$. This means we want to find a simple, smooth mathematical function that $r_k(N)$ gets closer and closer to as N becomes very large.

The final image shows a graph of what this might look like. The wavy line represents the actual, somewhat erratic value of $r_k(N)$ for different N. The smooth, dashed line is the theoretical asymptotic formula we are trying to find. The large question mark signifies that this formula is still unknown and is one of the great unsolved problems in combinatorics, as highlighted by Paul Erdős's significant prize offers.



The Goal: Find a formula that $r_k(N)$ approaches as N grows

In summary, the problem is to find a formula that describes the maximum size of a set of numbers within a given range that avoids any k-term arithmetic progression, as this range becomes infinitely large. This is a notoriously difficult problem with a rich history and is still an active area of research.